7 The Minimal Supersymmetric Standard Model: Part I

7.1 Particles

			$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
Q_i	$(\widetilde{u}_L, \widetilde{d}_L)_i$	(u_L, d_L)	$\Box$	$\Box$	$\frac{1}{6}$
$\overline{u}_i$	$\widetilde{u}^*_{Ri}$	$u^\dagger_{Ri}$	$\overline{\Box}$	$\mathbf{1}$	$-\frac{2}{3}$
$\overline{d}_i$	$\widetilde{d}^*_{Ri}$	$d^\dagger_{Ri}$	$\overline{\Box}$	$\mathbf{1}$	$\frac{1}{3}$
L_i	$(\widetilde{\nu}, \widetilde{e}_L)_i$	$(\nu, e_L)_i$	$\mathbf{1}$	$\Box$	$-\frac{1}{2}$
$\overline{e}_i$	$\widetilde{e}^*_{Ri}$	$e^\dagger_{Ri}$	$\mathbf{1}$	$\mathbf{1}$	1
H_u	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	$\mathbf{1}$	$\Box$	$\frac{1}{2}$
H_d	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	$\mathbf{1}$	$\Box$	$-\frac{1}{2}$
G	G^a_μ	$\widetilde{G}^a$	$\mathbf{Ad}$	$\mathbf{1}$	0
W	W^0_μ, $W^\pm_\mu$	$\widetilde{W}^0$, $\widetilde{W}^\pm$	$\mathbf{1}$	$\mathbf{Ad}$	0
B	B_μ	$\widetilde{B}$	$\mathbf{1}$	$\mathbf{1}$	0

Where

$$u_i = (u, c, t) \tag{7.1}$$
$$d_i = (d, s, b) \tag{7.2}$$
$$\nu_i = (\nu_e, \nu_\mu, \nu_\tau) \tag{7.3}$$
$$e_i = (e, \mu, \tau) \tag{7.4}$$

Two Higgs doublets with opposite hypercharges are needed to cancel the Y^3 and YW^2 gauge anomalies. In addition we need an even number of fermion doublets to avoid the Witten anomaly for $SU(2)_L$.

The superpotential is taken to be:

$$W = \overline{u}\mathbf{Y_u}QH_u - \overline{d}\mathbf{Y_d}QH_d - \overline{e}\,\mathbf{Y_e}LH_d + \mu H_u H_d \tag{7.5}$$

$SU(2)_W$ indices are contracted by $\epsilon^{\alpha\beta}$. Holomorphy requires both H_u and H_d in order to write Yukawa couplings for both u and d type quarks

For $m_t \gg m_c, m_u$; $m_b \gg m_s, m_d$; $m_\tau \gg m_\mu, m_e$

$$\mathbf{Y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} \tag{7.6}$$

$$
\mathbf{Y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} \tag{7.7}
$$

$$
\mathbf{Y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \tag{7.8}
$$

Then

$$
\begin{aligned} W &= y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ &\quad - y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+H_d^- - H_u^0H_d^0) \end{aligned} \tag{7.9}
$$

The μ-term gives a mass to the Higgs and Higgsinos

$$
\begin{aligned} \mathcal{L}_{\mu,\text{quadratic}} &= -\mu(\widetilde{H}_u^+\widetilde{H}_d^- - \widetilde{H}_u^0\widetilde{H}_d^0) + h.c. \\ &\quad -|\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2) \end{aligned} \tag{7.10}
$$

Thus there is a stable minimum at the origin. To have electroweak symmetry breaking we have to have soft SUSY breaking terms. Since the μ-term is supersymmetric there is no reason why it can't be as big as the Planck scale. In order to get the weak scale correctly without unnatural cancellations we will need $\mu \sim \mathcal{O}(m_{\text{soft}})$ rather than M_{Pl}. This is the μ-problem. A class of solutions takes μ to be forbidden at tree-level and then determined by the SUSY breaking which also determines m_{soft}. In the MSSM, μ is just another parameter.

$$
\begin{aligned} \mathcal{L}_{\mu,\text{trilinear}} &= \mu^* \Big(\widetilde{\overline{u}}\mathbf{Y_u}\widetilde{u}H_d^{0*} + \widetilde{\overline{d}}\mathbf{Y_d}\widetilde{d}H_u^{0*} + \widetilde{\overline{e}}\mathbf{Y_e}\widetilde{e}H_u^{0*} \\ &\quad + \widetilde{\overline{u}}\mathbf{Y_u}\widetilde{d}H_d^{-*} + \widetilde{\overline{d}}\mathbf{Y_d}\widetilde{u}H_u^{+*} + \widetilde{\overline{e}}\mathbf{Y_e}\widetilde{\nu}H_u^{+*}\Big) + h.c. \end{aligned} \tag{7.11}
$$

7.2 R-Parity

There are other holomorphic renormalizable terms we could put in the superpotential:

$$
W_{\text{disaster}} = \alpha^{ijk}Q_iL_j\overline{d}_k + \beta^{ijk}L_iL_j\overline{e}_k + \gamma^i L^i H_u + \delta^{ijk}\overline{u}_i\overline{u}_j\overline{d}_k \tag{7.12}
$$

W_{disaster} violates Lepton and Baryon number!

$$
\Gamma \approx \frac{|\alpha\delta|^2}{m_{\tilde{q}}^4}\frac{m_p^5}{16\pi^2} \tag{7.13}
$$

$$
\tau = \frac{1}{\Gamma} \approx \frac{1}{|\alpha\delta|^2}\left(\frac{m_{\tilde{q}}}{1\,\text{TeV}}\right)^4 10^{-10}\,\text{s} \tag{7.14}
$$

Experimentally $\tau_p > 10^{32}$ years $\approx 3 \times 10^{39}$ s. So we need $|\alpha\delta| < 2 \times 10^{-25}$.

One solution is to invent a new symmetry called R parity:

$$\begin{array}{ccc} \text{(observed particle)} & \rightarrow & \text{(observed particle)} \\ \text{(superpartner)} & \rightarrow & -\text{(superpartner)} \end{array} \tag{7.15}$$

R parity forbids W_{disaster}. The name comes from the fact that it can arise as a discrete Z_2 subgroup of a $U(1)_R$ symmetry. The full $U(1)_R$ symmetry is too restrictive since it forbids gaugino mass terms. R parity can also arise as a discrete subgroup of $B - L$

$$R = (-1)^{3(B-L)+F} \tag{7.16}$$

R parity is part of the definition of the MSSM. There are of course alternatives that give non-minimal SSM's.

R parity has important phenomenological consequences:

- at colliders superpartners are produced in pairs
- the lightest superpartner (LSP) is stable, and thus (if it is neutral) can be a dark-matter candidate
- each sparticle (aside from the LSP) eventually decays into an odd number of LSP's.

7.3 Soft Breaking

$$\begin{aligned} \mathcal{L}_{\text{soft}}^{\text{MSSM}} &= -\frac{1}{2}\left(M_3 \widetilde{G}\widetilde{G} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B}\right) + h.c. \\ &\quad -\left(\widetilde{\overline{u}}\,\mathbf{A_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{A_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{A_e}\,\widetilde{L}H_d\right) + h.c. \\ &\quad -\widetilde{Q}^*\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^*\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}^*\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}} - \widetilde{\overline{d}}^*\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}} - \widetilde{\overline{e}}^*\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}} \\ &\quad - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + h.c.)\,. \end{aligned} \tag{7.17}$$

In the literature b is often called $B\mu$. As we have seen all these parameters should be related to $m_{\text{soft}} \approx 1$ TeV in order to solve the hierarchy problem:

$$M_i, \mathbf{A_P} \sim m_{\text{soft}} \tag{7.18}$$

$$\mathbf{m_P^2}, b \sim m_{\text{soft}}^2 \tag{7.19}$$

With all these soft SUSY breaking terms the MSSM has 105 more parameters than the SM! This means that there are very few unambiguous predictions of the MSSM.

References

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